Summer Enrichment Packet

Rising Foundations for Algebra Students

ANSWER KEY



PRINCE GEORGE'S COUNTY PUBLIC SCHOOLS
Division of Academics
Department of Curriculum and Instruction



Foundations For Algebra

Note to the Student

You learned so much in Grade 7! It is important that you keep practicing your math skills over the summer to be ready for the Accelerated 2 course. In this packet, you will find weekly activities for the summer break.

Directions:

- ➤ Create a personal and fun math journal by stapling several pieces of paper together or use a notebook or binder with paper. Be creative and decorate the cover to show math in your world.
- Each journal entry should:
 - Have the week number and the problem number.
 - ❖ Have a clear and complete answer that explains your thinking.
 - ❖ Be neat and organized.
- Pay attention to the gray boxes that you see at the beginning of each week's activities. Those boxes indicate the Common Core domain and standard that the

subsequent activities address. If you see a NON-CALCULATOR SYMBOL to a gray box, then <u>do not use a calculator for the activities in that section!</u>

Playing board games and card games are a good way to reinforce basic computation skills and mathematical reasoning. Try to play board and card games at least once a week. Some suggested games to play are: Monopoly, Chess, War, Battleship, Mancala, Dominoes, Phase 10, Yahtzee, 24 Challenge, Sudoku, KenKen, Connect Four and Risk.

Foundations For Algebra Where to Go to Get Help and Opportunities for Practice!

During the course of your math work this summer, you may need some assistance with deepening your understanding the skills and concepts. You also might want to get some more practice. Here are some sites you can visit online:



To get the exact definition of each standard, go to www.corestandards.org and search for the content standard (for example, 7.NS.1a).



Khan Academy has helpful videos and self-guided practice problems for every grade level. Go to www.khanacademy.org to get started.

Foundations For Algebra

WEEK 1 || **Ratios and Proportional Relationships Standards 7.RP.1-7.RP.3:** Analyze proportional relationships and use them to solve real-world and mathematical problems.

The students in Ms. Brown's art class were mixing yellow and blue paint. She told them that two mixtures will be the same shade of green if the blue and yellow paint are in the same ratio.

The table below shows the different mixtures of paint that the students made.

	Α	В	С	D	E
Yellow	1 part	2 parts	3 parts	4 parts	6 parts
Blue	2 part	3 parts	6 parts	6 parts	9 parts

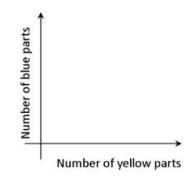
a. How many different shades of paint did the students make?

The students made two shades of paint.

b. Some of the shades of paint were bluer than others. Which mixture(s) were the bluest? Show work or explain how you know.

Mixtures A and C were bluer than B, D, and E because they contain a ratio of 2 parts blue to 3 total parts (\approx 67% blue) while B, D, and E have a ratio of 3 parts blue to 5 total parts (\approx 60% blue).

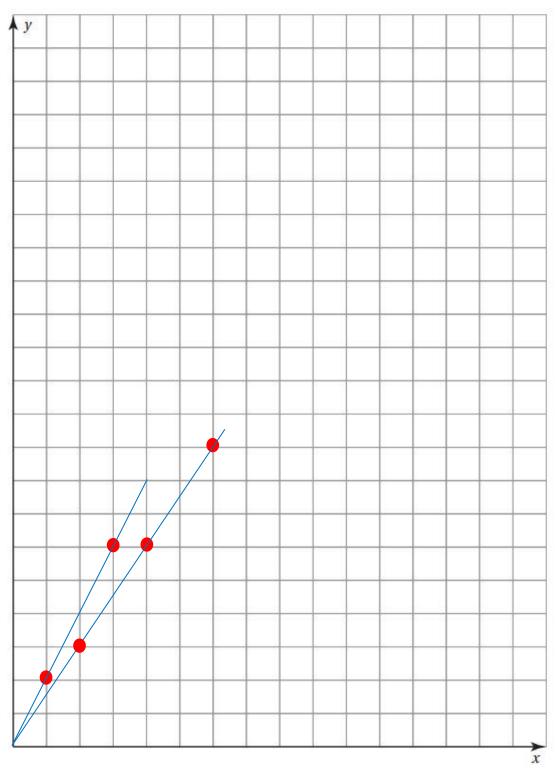
c. Using the coordinate grid on the next page, carefully plot a point for each mixture on a coordinate plane like the one that is shown in the figure to the right.



d. Draw a line connecting each point to (0,0). What do the mixtures that are the same shade of green have in common?

The mixtures that are the same shade of green lie on the same line that goes through (0,0). It shows that they have equivalent ratios.

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Directions: Complete the following three problems to apply your understanding of percentages and ratios.



Problem #1:

Jesse's Awesome Autos advertised a special sale on cars - Dealer cost plus 5%! Quinten and Shapera bought a luxury sedan for \$23,727.90. What was the dealer's cost?

Dealer's cost = d d + 0.05d = 23,727.901.05d = 23,727.90, so d = \$22,598

Problem #2:

You and some friends went out to T.G.I. Fridays for dinner. You ordered a root beer, sweet potato fries and cheese quesadillas. The total bill came to \$21.86. Your dad has told you many times that it's important to leave a good tip; about 20%. You have \$26.00 in your wallet. How much would the total be if you left a 20% tip? Can you cover the cost? $21.86 \times 1.20 = 26.23$. If I left a 20% tip, the total would be \$26.23, so \$26 is not quite enough to cover the cost of the meal and a 20% tip.

Problem #3:

Builders have observed that windows in a home are most attractive if they have the width to length ratio 3:5. If a window is to be 48 inches wide, what should its length be for the most attractive appearance?

It would be 80 inches in length because a width of $3 \times 16 = 48$; to find an equivalent ratio, multiply 5×16 to get 80.

- 2. Create your own problems.
 - Create one original problem involving a percentage (discount or tax).
 - Create one original problem involving a ratio or part/whole relationship.
 - Solve both and keep the answer key.
 - Challenge a friend or family member to solve your problems.

Answers will vary.

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WEEK 2 || **Ratios and Proportional Relationships Standards 7.RP.1-7.RP.3:** Analyze proportional relationships and use them to solve real-world and mathematical problems.

Mariah's family is driving to her grandmother's house. Her family travels 370.5 miles between 10:15 a.m. and 4:45 p.m.



Part A

What is an equation that Mariah can use to determine their average rate of travel for the day, R, in miles per hour?

Choose the correct numbers and operations from the boxes below. Explain your answer in the space below.

370.5	6.5	10.25	4.75
+	ı	•	•••

$$_{370.5}$$
 \div $_{6.5}$ = R

____The rate of miles per hour means "miles DIVIDED by hours. So if Mariah's family___

___ traveled 370.5 miles, then I would divide the miles by the hours.

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Part B

Calculate the family's average rate of travel for the day. Show your process for determining your answer. If I divide the miles, 370.5, by the hours, 6.5, I get 57. So Mariah's family traveled 57 miles per hour. Part C Mariah tells her family, "It's a good thing we traveled as fast as we did. If our rate had been 50 miles per hour, we wouldn't have gotten to his house until about..." Then show the process you used for determining your answer. Then fill in the blank to complete the statement at the bottom. If Mariah's car was going 50 miles per hour, then it would have taken them ___ ___ hours and her family would have gotten to the house at 5:40 p.m. _____ $370.5 \div 50 = 7.41$, which is 7 hours and 25 minutes. 5:40 p.m. is 7 hours and ___ 25 minutes after 10:15 a.m. _____ If their average rate had been 50 miles per hour, Mariah's family would have arrived

at her grandmother's house at <u>5:40</u> p.m.

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Spicy Vegetables!



A restaurant makes a special seasoning for all its grilled vegetables. Here is how the ingredients are mixed:

$\frac{1}{2}$ of the mixture is salt	$\frac{1}{4}$ of the mixture is pepper
$\frac{1}{8}$ of the mixture is garlic powder	$\frac{1}{8}$ of the mixture is onion powder

When the ingredients are mixed in the same ratio as shown above, every batch of seasoning tastes the same.

Part A

Study the measurements for each batch in the table. Fill in the blanks so that every batch will taste the same.

	Batch 1	Batch 2	Batch 3
Salt (cups)	1	2	4
Pepper (cups)	1/2	1	2
Garlic Powder (cups)	$\frac{1}{4}$	1/2	1
Onion Powder (cups)	1/4	1/2	1

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Part B

The restaurant mixes a 12-cup batch of the mixture every week. How many cups of each ingredient do they use in the mixture each week? Show your work in the space below.

_Answers will vary
In Batch 2, the ingredients total 4 cups $(2 + 1 + \frac{1}{2} + \frac{1}{2})$. I know that $4 \times 3 = 12$,
so I can multiply each of the quantities of ingredients by 3 to get the correct
amounts for 12 cups:

 $2 \times 3 = 6$ cups of salt

 $1 \times 3 = 3$ cups of pepper

 $\frac{1}{2}$ x 3 = 1.5 cups of garlic powder

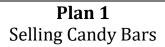
 $\frac{1}{2}$ x 3 = 1.5 cups of onion powder

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WEEKS 3 and 4 | Expression and Equations Standards 7.EE.3-7.EE.4: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Choosing a School Fund-Raiser

A school is going to have a fundraiser to buy new books for the library. The goal is to raise at least \$1000. Three different fundraising plans are being discussed.





Plan 2 **Selling Carnations**



Plan 3

Holding a Walkathon



In order to evaluate the three plans, you will need to answer the following questions about each plan.

Plan 1: Selling Candy Bars

Explain or show your reasoning as you answer the questions below. You may use a combination of diagrams, drawings, expressions/equations, and words.



The school is able to buy 6 boxes of candy bars for \$36.80. Each box contains 24 candy bars. What is the cost per candy bar? 36.80 / 6 = 6.13 per box.

6.13 / 24 = approx. \$0.26 per bar

Each candy bar will be sold for \$2.00. What is the minimum number of candy bars that must be sold to meet the goal of raising at least \$1000? (Amount raised = Earnings *minus* costs)

2.00 - 0.26 = 1.74 profit per bar

1000 / 1.74 = 575 (rounded)

The students must sell a minimum of 575 candy bars to meet the goal.

The goal is to have 150 students in the school sell candy bars for the fundraiser. On average, how many candy bars must be sold per student to meet the goal?

575 / 150 = 3.83

Each student must sell about four candy bars to meet the goal.

Plan 2: Selling Flowers

In answering each question, explain or show your reasoning. You may use a combination of diagrams, drawings, expressions/equations, and words.

The school is able to buy a dozen carnations for \$8.68. For the fundraiser, the carnations will be sold with a 90% markup. For what price will the school sell 1 dozen carnations?

 $8.68 \times 1.9 = 16.49$

The school will sell 1 dozen carnations for 16.49.



The school will be charged a one-time shipping fee of \$89.50 for the flowers. For each flower sold, the school will earn \$1.05 for the fundraiser.

Explain why both of these inequalities could be used to determine the number of carnations the students need to sell to meet the goal of \$1000:

- $1.05n 89.5 \ge 1000$, where n represents the number of carnations sold
- 1.05(12d) $89.5 \ge 1000$, where *d* represents the number of dozens of carnations sold.

If I solve for *n* in the first inequality, I get about 1,038.

If I solve for *d* in the second inequality, I get about 86.

This makes sense because the second inequality deals with *dozens* of carnations, so to convert to individual carnations, you would multiply by 12, and $86 \times 12 \approx 1,038$

Use the following inequality to determine n, the minimum number of carnations the students need to sell to meet the goal.

 $1.05n - 89.5 \ge 1000$

 $n \ge 1.038$

The students at the school must sell at least 1,038 carnations to meet the goal.

The goal is to have 150 students sell carnations for the fundraiser. If each student sells the same number of carnations, approximately how many carnations will each student sell?



1038 / 150 = 6.92

Each student must sell about 7 carnations.

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Plan 3: Walkathon

The third possible fundraiser is a walkathon. Each lap around a track is $\frac{1}{4}$ of a mile. Students will receive a donation for each lap they walk around the track. The principal expects each student to walk $1\frac{1}{2}$ laps in $\frac{1}{4}$ of an hour.



In answering each question, explain or show your reasoning. You may use a combination of diagrams, drawings, expressions/equations, and words.

To meet the principal's expectation, at what rate of miles per hour must the students walk?

1.5 laps in 0.25 hours is the same as 6 laps in 1 hour (after multiplying each part of the ratio by 4. If each lap is 0.25 of a mile, then $0.25 \times 6 = 1.5$ miles per hour.

The fundraiser will require the students to walk 9 complete laps. If a student meets the principal's expectation, how many hours will it take to walk 9 complete laps?

If a student walks 1.5 laps in 0.25 hours, that is 3 laps in 0.5 hours.

 $3 \operatorname{laps} x 3 = 9 \operatorname{laps}$

0.5 hours x 3 = 1.5 hours

It will take a student 1.5 hours to walk 9 laps.

Each student will receive \$1.50 per lap. If each student completes exactly 9 laps, what is the minimum number of students that will be needed to meet the goal of raising at least \$1000?

9 laps x 1.50 = 13.50

1000 / 13.50 = 74.07

A minimum of 75 students would need to walk in order to meet the goal of raising \$1,000.

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Conclusion: Based on your analysis, which fundraising plan would you recommend the school use? Include any relevant information and mathematical reasoning to support your answer.

Answers will vary.

I would recommend the walk-a-thon because only 75 students would need to walk 9 laps each in order to meet the fundraising goal. This option does not involve selling anything and it would require only half of the students to participate compared to the other fundraiser options.

<u>WEEK 5</u> || **Expression and Equations Standards 7.EE.3-7.EE.4:** Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Reviewing Steps for Solving Equations and Properties of Equality

Addition Property of Equality

Words Adding the same number to each side of an equation produces an equivalent equation.

Algebra If a = b, then a + c = b + c.

Subtraction Property of Equality

Words Subtracting the same number from each side of an equation produces an equivalent equation.

Algebra If a = b, then a - c = b - c.

If
$$a = b$$
, then $a + c = b + c$

If a = b, then

a - c = b - c

Multiplication Property of Equality

Words Multiplying each side of an equation by the same number produces an equivalent equation.

Algebra If a = b, then $a \cdot c = b \cdot c$.

Division Property of Equality

Words Dividing each side of an equation by the same number produces an equivalent equation.

Algebra If a = b, then $a \div c = b \div c$, $c \ne 0$.

If
$$a = b$$
, then

$$a \cdot c = b \cdot c$$

If a = b, then

$$a \div c = b \div c$$

Turning a TV *on* and turning that TV *off* can be considered *inverse operations*. Describe two other real-life situations that can be thought of as inverse operations.

__Answers will vary. _____

Describe the inverse operation that will undo the given operation:

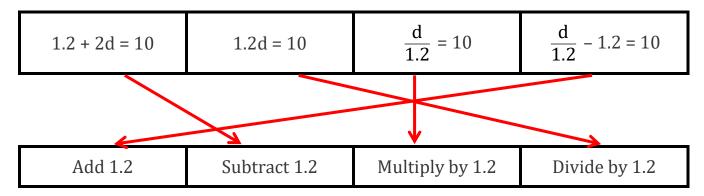
Multiplying by \frac{2}{7} ___Dividing by 2/7 or multiplying by 7/2_____

Subtracting -4 __Adding -4 or subtracting 4_____

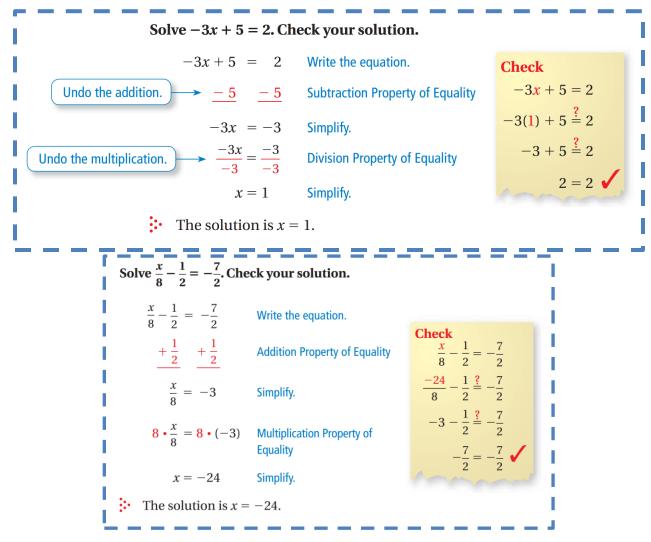
Dividing by -3.5 __Multiplying by -3.5 ____

Adding $\frac{2}{3}$ __Subtracting $\frac{2}{3}$

Match the equation in the top table with the first step to solve it in the bottom table.



Review the two examples that show the process for solving an equation below. Make sure to attend to any properties of equality shown next to solution steps.



Write the properties that correspond to each solution step for the equation. Choose from the following properties and actions:

Addition Property of Equality Simplify

Subtraction Property of Equality Distributive Property

Multiplication Property of Equality Collect Like Terms

Division Property of Equality

$$\frac{m}{2} + 6 = 10$$

$$\frac{m}{2} + 6 - 6 = 10 - 6$$

$$\frac{m}{2} = 4$$

$$2 \cdot \frac{m}{2} = 2 \cdot 4$$

$$m = 8$$
Subtraction Prop of Equality

Multiplication Prop of Equality

Simplify

Simplify

Simplify

$$6(x-2) = -18$$

$$6 \cdot x - 6 \cdot 2 = -18$$

$$6x - 12 = -18$$

$$6x - 12 + 12 = -18 + 12$$

$$6x = -6$$

$$\frac{6x}{6} = \frac{-6}{6}$$

$$x = -1$$

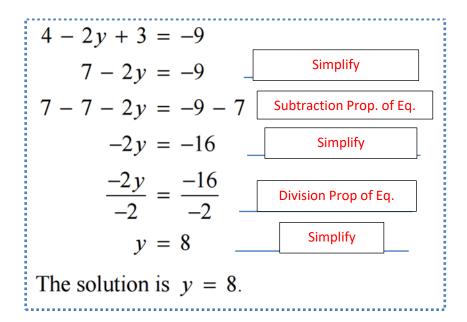
The solution is $x = -1$.

Distributive Property

Addition Prop of Eq.

Simplify

Division Prop of Eq.



For each of the equations above, use the given solution to check the equation. Show your work in the space below.

$\frac{m}{2} + 6 = 10$ $-8/2 = 4$ $-4 + 6 = 10$	6(x-2)=-186 * -3 = -18	4-2y+3=-92 * 8 = 164 - 16 = -1212 + 3 = -9
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<u>WEEK 6</u> || Expression and Equations Standards 7.EE.3-7.EE.4: Solve reallife and mathematical problems using numerical and algebraic expressions and equations.



Identifying and Correcting Errors in Solutions

Nancy thought she correctly solved the equations below, but she made some mistakes. Examine her work below. What should Nancy do to correct the errors that she made? Use the second column in the table to *Identify and Explain Her Errors* and use the third column to indicate the *Correct Steps and Correct Solution*.

Nancy's Work	Identify and Explain	Correct Steps and
	Her Error(s)	Correct Solution
$ 8-5c = -37 \\ -8 -5c = -37 \\ -8 -8 \\ \hline 5c = -45 \\ 5 -9 $	After she subtracted 8 from both sides of the equation, she should have had -5c = -45 on the next line. She left off the negative sign on the 5c term.	8 - 5c = -37 8 - 5c - 8 = -37 - 8 -5c = -45 -5c/-5 = -45/-5 c = 9
3(2x-4) = 8 $3(2x-4) = 8$ $6x-4 = 8$ $+4 + 4$ $6x = 12$ 6 $x = 2$	When Nancy distributed the factor 3 to each of the terms in the parentheses, she did not multiply the 3 by the (- 4). So on the next line, she should have had 6x - 12.	3(2x - 4) = 8 6x - 12 = 8 6x - 12 + 12 = 8 + 12 6x = 20 6x/6 = 20/6 x = 3 and $1/3$
3x + 2x - 6 = 24 $-2x - 2x$ $x - 6 = 24$ $-2x - 2x$ $x - 6 = 24$	Nancy mistakenly subtracted 2x twice from the same side of the equation. She should have combined the x terms first and then solved the equation.	3x + 2x - 6 = 24 5x - 6 = 24 5x - 6 + 6 = 24 + 6 5x = 30 5x/5 = 30/5 x = 6

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Nancy's Work	Identify and Explain	Correct Steps and		
	Her Error(s)	Correct Solution		
-2(x-2) = 14 $-2(x-2) = 14$ $-2x-4 = 14$ $+4 + 4$ $-2x = 18$ $-2 = 2$ $x = -9$	When Nancy distributed the (-2) term, she correctly multiplied the (-2) by the x term but instead of multiplying the -2 by the (- 2) and getting + 4, she wrote - 4.	-2(x - 2) = 14 $-2x + 4 = 14$ $-2x + 4 - 4 = 14 - 4$ $-2x = 10$ $-2x/-2 = 10/-2$ $x = -5$		
3(2x+1) + 4 = 10 $3(2x+1) + 4 = 10$ $6x + 3 + 4 = 10$ $9x + 4 = 10$ $-4 - 4$ $9x = 6$ $4x + 6$ $9x = 6$ $4x = 6$ $4x$	After Nancy correctly distributed the 3 to each term, she incorrectly combined the 6x and 3 terms. She can't do that because they are not like terms.	3(2x + 1) + 4 = 10 6x + 3 + 4 = 10 6x + 7 = 10 6x + 7 - 7 = 10 - 7 6x = 3 6x/6 = 3/6 x = 1/2		

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Which is the Odd One Out?

Use your knowledge of properties and equations to determine which equation is *not* equivalent to the other three equations in the row.

$$2.5x = 12.5$$

$$22.5 = 10 + 2.5x$$

$$5(0.5x + 2) = 22.5$$

$$0.5x + 5 = 12.5$$

solution: x = 5 solution: x = 5 solution: x = 15

 $\frac{1}{2}(a+24)=7$

$$-5=\frac{1}{2}a$$

$$7=12+\frac{1}{2}a$$

$$\frac{1}{2}a + 12 = -5$$

_solution: a = -10 _____solution: a = -10 _____solution: a = -10 _____solution: a = -34_____

-9 = 5 - 3c + 4

$$3c - 9 = -9$$

$$-4c + c + 9 = -9$$

$$-3(c-3)=-9$$

_solution: c = 6 _____solution: c = 0 ____solution: c = 6 ____solution: c = 6 ____

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<u>WEEK 7</u> || Ratios and Proportional Relationships Standards 7.RP.2: Recognize and represent proportional relationships between quantities.

Proportional Relationships and Constant of Proportionality

Proportional relationships are two-variable relationships that have a consistent rate of change and include the value (0, 0). Proportional relationships can be shown in a variety of ways – in words, a table, a graph, and an equation.

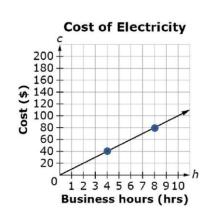
In a proportional relationship, the rate of change, expressed as a ratio of the *dependent* variable to the *independent* variable, yields the *constant of proportionality*. The constant of proportionality can be written as a constant in an equation in the form of y = cx, where c is the constant of proportionality.

Review the four representations of a proportional relationship shown in the table below.

Words

A company is tracking its electricity use and charges on a given day. After 4 hours, it has accumulated \$40 in charges. After 8 hours, it has accumulated \$80 in charges.

Graph on Coordinate Grid



Table

Hours (h)	Cost (c)
2	20
4	40
8	80

Equation

The rate of change is \$10 for every hour. So the equation is:

$$c = 10h$$

Foundations For Algebra Proportional Relationships / Constant of Proportionality Sort

Identify which graphs and tables do or do not show a *proportional relationship* by placing the letter of each graph or table in the corresponding column. For the tables or graphs that show proportional relationships, determine the *constant of proportionality*.

х у	х у	x y	х у	х у
-2 4	2 2	1 2	2 3	4 2
-1 2	3 4	2 3	4 6	8 4
0 0	4 6	3 4	6 9	12 6
1 2	5 8	4 5	8 12	16 8
Α	В	С	D	E
0 1 2 3 4 5 6 7 8 9 10	-4 -3 -2 -1 -1 -3 4 5 6 x	Cost of Mailing a Package Cost of Mailing a Package Discontinuous and the second sec	300 82 250 82 250 83 250 84 8 12 16 20 24 Number of Boxes	(Sample of packs) (Sample of packs)

Does Not Show a Proportional Relationship	Shows a Proportional Relationship	What is the Constant of Proportionality?
Α	D	3/2
В	E	1/2
С	I	12.5
F	J	2
G		\Rightarrow
Н		\Rightarrow

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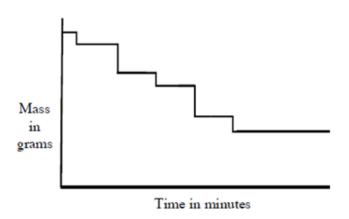
<u>WEEK 8</u> || **Functions Standards 8.F.4-8.F.5:** Use functions to model relationships between quantities.

Graph #1: Snacking on M&Ms

Kelly is eating a pack of M&Ms.

The graph shows the changes in the mass of the M&Ms in the pack as time passes.





1. What is Kelly doing when there is a vertical line on the graph?

__Removing M&M's from the bag______

2. Why are the vertical lines of different lengths?

_The shorter vertical lines indicate when she took out fewer M&Ms at a time. The longer lines are for when she was taking out more M&Ms._____

3. Did Kelly eat all of the M&Ms? Explain how you know.

___Kelly did not eat all of the M&Ms. The graph line does not reach the x-axis, _____

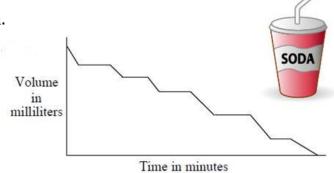
_and if she did eat all of them, then the graph line would have reached it (meaning zero grams of M&Ms in mass.)

Graph #2: Drinking a Cup of Soda

Ezra is drinking with a straw from a cup of soda.

The graph shows the volume of soda in the cup as time passes.

1. What is happening when the line on the graph is horizontal?



__Horizontal lines indicate that Ezra is not drinking from the cup. _____

2. Why do the lines going downwards on this graph go at an angle?

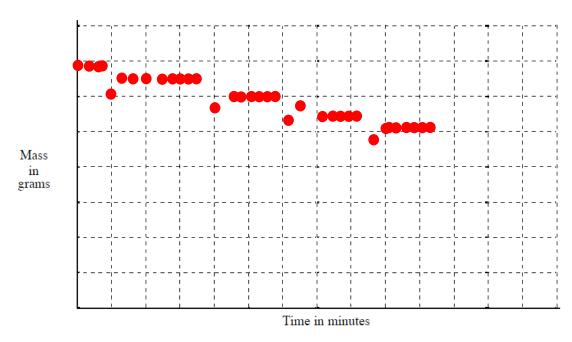
___Ezra is gradually drinking the soda, so the slanted lines show that he is taking a sip____

_over a span of a few seconds. _____

Graph #3: Eating Cherries

Kara is eating cherries from a bag. After eating a cherry, she puts the stone back into the bag before taking out the next cherry.

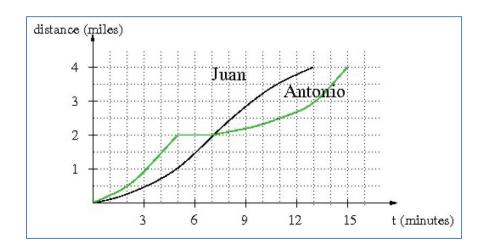
1. On the grid, draw a graph to show the changes in the mass of the bag of cherries as time passes.



Foundations For Algebra

Graph #4: Bike Race

Antonio and Juan are in a 4-mile bike race. The graph below shows the distance of each racer (in miles) as a function of time (in minutes).



- 1. Who wins the race? How do you know? _Juan won the race because it took him 13 minutes to go 4 miles and it took Antonio 15 minutes to go 4 miles
- 2. Give a possible explanation for Antonio's progress during minutes 5 to 7 of the race.

Antonio may have gotten injured or gotten a flat tire and had to stop during minutes

_5 to 7 of the race.____

3. Imagine you were watching the race and had to announce it over the radio. Write a short story describing what happened in the race.

___Stories will vary. _____

__Story should include that Antonio led the race in the early part; a reason why Antonio did not progress during minutes 5 to 7; and that Juan passed Antonio at minute 7 and won the race.
