Summer Enrichment Packet for Rising Algebra 2 Students

PRINCE GEORGE'S COUNTY PUBLIC SCHOOLS
Division of Academics
Department of Curriculum and Instruction

Summer Student Enrichment Packet for Rising Algebra 2 Students

Note to Student: You've learned so much in Algebra I and Geometry! It is important that you keep practicing your mathematical knowledge over the summer to prepare yourself for Algebra II. In this packet, you will find weekly activities for the Summer Break.

Directions:

- Create a personal and fun math journal by stapling several pieces of paper together or use a notebook or binder with paper. Be creative and decorate the cover to show math in your world.

- Each journal entry should:
  - Have the week number and the problem number.
  - Have a clear and complete answer that explains your thinking.
  - Be neat and organized.

Playing board and card games are a good way to reinforce basic computation skills and mathematical reasoning. Try to play board and card games at least once a week. Some suggested games to play are: Monopoly, Chess, War, Battleship, Mancala, Dominoes, Phase 10, Yahtzee, 24 Challenge, Sudoku, Connect Four, and Risk.

Where to Go to Get Help and Opportunities for Practice!

During the course of your math work this summer, you may need some assistance with deepening your understanding the skills and concepts. You also might want to get some more practice. Here are some sites you can visit online:

COMMON CORE STATE STANDARDS INITIATIVE PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER	To get the exact definition of each standard, go to www.corestandards.org and search for the content standard (for example, *HSF.BF.1.3*).
KHAN ACADEMY	***Khan Academy*** has helpful videos and self-guided practice problems for every grade level. Go to www.khanacademy.org to get started.

Week 1

Domain: *Algebra*

Standards:

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Jesse's Phone Plan

Jesse has a prepaid phone plan (Plan A) that charges 15 cents for each text sent and 10 cents per minute for calls.

1. If Jesse uses only text, write an equation for the cost $\boldsymbol{C}$ of sending $\boldsymbol{t}$ texts.

a. How much will it cost Jesse to send 15 texts? Justify your answer.

b. If Jesse has $6, how many texts can he send? Justify your answer.

2. If Jesse only uses the talking features of his plan, write an equation for the cost $\boldsymbol{C}$ of talking $\boldsymbol{m}$ minutes.

a. How much will it cost Jesse to talk for 15 minutes? Justify your answer.

b. If Jesse has $6, how many minutes can he talk? Justify your answer.

3. If Jesse uses both talk and text, write an equation for the cost $\boldsymbol{C}$ of sending $\boldsymbol{t}$ texts and talking $\boldsymbol{m}$ minutes.

a. How much will it cost Jesse to send 7 texts and talk for 12 minutes? Justify your answer.

b. If Jesse wants to send 21 texts and only has $6, how many minutes can he talk? Justify your answer.

c. Will sending 21 texts use all of his money? If not, how much money will he have left? Justify your answer.

4. Jesse discovers another prepaid phone plan (Plan B) that charges a flat fee of $15 per month, then $.05 per text sent or minutes used. Write an equation for the cost of Plan B.

5. In an average month, Jesse sends 200 texts and talks for 100 minutes. Which plan will cost Jesse the least amount of money? Justify your answer.

Week 2

Domain: *Algebra; Number and Quantity*

Standards:
N-Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
A-CED.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
A-CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
A-REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

IVY CARTER GROWS UP

You are a medical assistant in a pediatrician's office and one of your responsibilities is evaluating the growth of newborns and infants. Your first patient, a baby girl named Ivy Carter, was 21.5 inches long at 3 months old. At 8 months, you measure her at 24 inches long. For your medical records, all measurements must be given both in inches and in centimeters: 1 inch = 2.54 cm

1. Assuming Ivy's growth is linear, find a linear model for her growth (in inches) over time (in months).

2. Use your model to determine how long Ivy was at birth (in centimeters)? Explain how you know your answer is correct, assuming this model.

3. Use your model to determine approximately how tall Ivy will be at 1 year old. At 3 years old (in centimeters). Show how you know your answers are correct.

4. Use your model to estimate how old Ivy will be (in years and months) when she measures at 48 inches. Show how you know your estimate is accurate.

5. Complete the table below and use the chart to plot Ivy Carter's growth, based on the calculations above (record # 1234.56).[1]

Age (months)	Length (in)	Length (cm)
0		
3	21.5	
8	24	
12		
36		
	48	

6. What is Ivy's approximate length-for-age percentile at each of these ages?

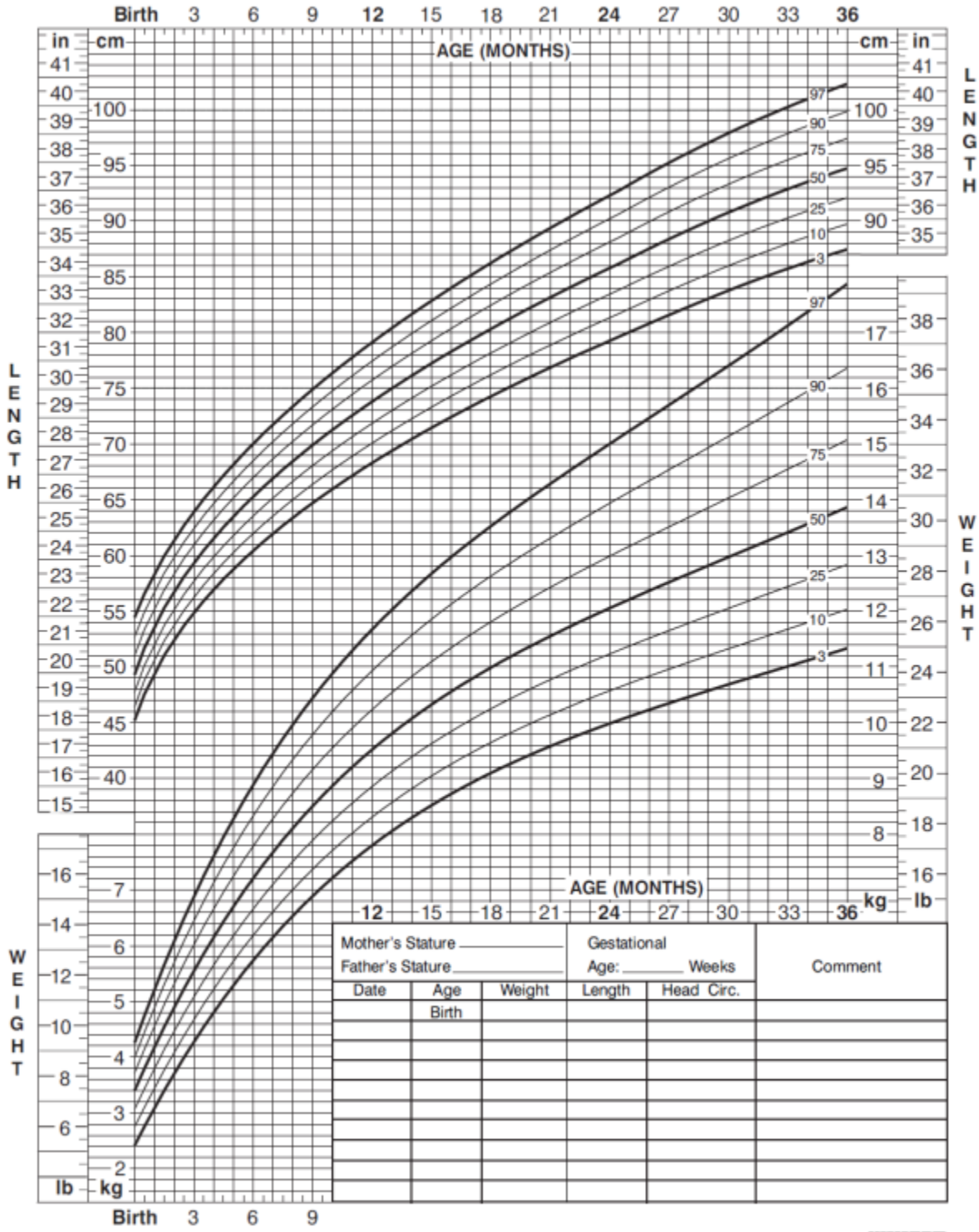
Age (months)	Length (in)	Length (cm)	Percentile
0			
3	21.5		
8	24		
12			
36			
	48		

[1] http://www.cdc.gov/growthcharts/charts.htm#Set1

Birth to 36 months: Girls
Length-for-age and Weight-for-age percentiles

NAME _____

RECORD # _____



Mother's Stature _____ Father's Stature _____			Gestational Age: _____ Weeks		Comment
Date	Age	Weight	Length	Head Circ.	
	Birth				

Published May 30, 2000 (modified 4/20/01).
SOURCE: Developed by the National Center for Health Statistics in collaboration with the National Center for Chronic Disease Prevention and Health Promotion (2000).
http://www.cdc.gov/growthcharts

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Week 3

Domain: *Algebra; Number and Quantity*

Standards:
N-Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
A-CED.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
A-REI.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7.RP3: Use proportional relationships to solve multistep ratio and percent problems.

DENTAL IMPRESSIONS

To fabricate a stone model from a dental impression, you need 30 mL of water to 100 grams of gypsum powder. In your orthodontic office, in an average week, you make 75 impressions. It is most cost effective to order the powder in 50-lb boxes. (Remember: 1 gram = .0022 pounds)

1. How many stone models can you make with one 50-lb box? Show your work and mathematical thinking.

2. How frequently will you need to reorder your powder supply? Show your work and mathematical thinking.

3. What is your office's annual demand for the gypsum powder? Show your work and mathematical thinking.

4. Your office is considering purchasing digital impression technology, but only if it proves to be more cost effective within two years. Assuming the initial investment in technology and training for the digital impression scanner is $115,000, the technology will eliminate the number of manual/non-digital impressions by 60%, and manual impressions cost your office $30 on average, create equations to determine your break-even point on this investment.
Support your solution both algebraically and graphically.

5. Write a recommendation to your office encouraging or discouraging the purchase of the digital impression technology based on your findings above.

Week 4

Domain: *Algebra; Number and Quantity*

Standards:
A.REI.6 Solve systems of linear equations exactly and approximately focusing on pairs of linear equations in two variables.
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*
N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*

Where's the Beef?

On average, a grocer sells the following amounts of ground beef each week:

- 485 lbs of Regular Ground Beef (25% fat)
- 1,010 lbs of Lean Ground Beef (18% fat)
- 537 lbs of Extra Lean Ground Beef (12% fat)

He orders boneless round and lean trim beef products from which he produces his own ground beef. He orders:

Beef Type	% Lean	Price per Pound
Boneless Round	95%	$2.13
Lean Trim	70%	$1.82

Fill in the following table as you answer questions 1 and 2 below:

Type	Regular Ground Beef (lbs)	Lean Ground Beef (lbs)	Extra Lean Ground (lbs)	TOTALS
Boneless Round				
Lean Trim				
TOTALS				

Show <u>all</u> your work:

1. How many pounds of *boneless round* and *lean trim* are needed to make each of the ground beef mixtures to meet demand?

2. How many total pounds of each type will he need to buy each week?

3. To make a 15% profit, how much does he have to charge per pound for each ground beef mixture?

4. On which product is he making the higher profit and how much is that profit?

5. What is his total profit?

6. The grocer has the option to purchase Bull Meat, which is 92% lean and costs $2.07 per pound. If the grocer replaces the Boneless Round with the Bull Meat, how much will his total profit change, assuming that the charge per pound (calculated in question 3) remains the same?

7. A random sample of 376 customers reveals that 127 bought Extra Lean Ground Beef; how many in the next 500 customers could be expected to purchase Extra Lean Ground Beef.

Week 5

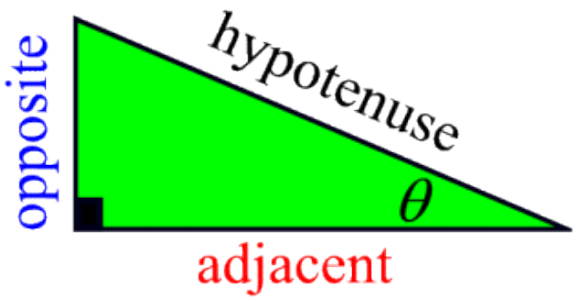
Domain: *Geometry*

Standards:
G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.
G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

Right Triangle Trigonometry

Directions: Study the information below.

In a right triangle, there are actually six possible trigonometric ratios, or functions. A Greek letter (such as theta θ or phi φ) will now be used to represent the angle.



sine of $\theta = \sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	cosecant of $\theta = \csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$
cosine of $\theta = \cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	secant of $\theta = \sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$
tangent of $\theta = \tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$	cotangent of $\theta = \cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

Notice that the three new ratios at the right are reciprocals of the ratios on the left. Applying a little algebra shows the connection between these functions.

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\dfrac{\text{opposite}}{\text{hypotenuse}}} = \frac{1}{\sin\theta}$$

Use the information in the chart to answer the following questions:

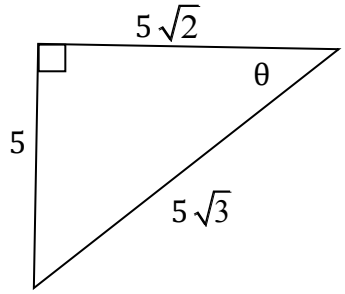
1. If $\sin(\theta) = \frac{4}{5}$, what is the $\csc(\theta)$? Explain your reasoning.

2. How do you find a side length and angle measure in a right triangle?

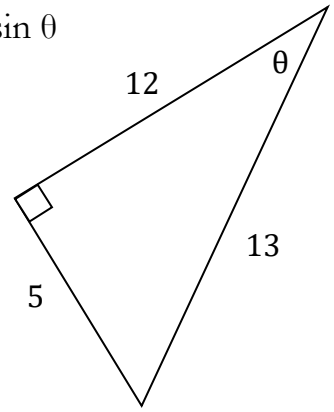
3. Given one trigonometric ratio, how would you find the other five ratios?

Use the right triangles to find the given values. Do not simplify.

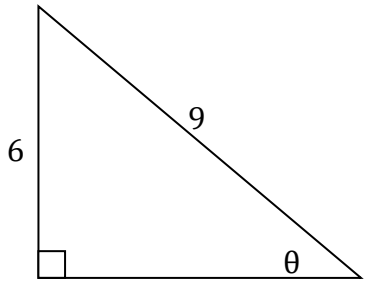
4. Find $\cos\theta$



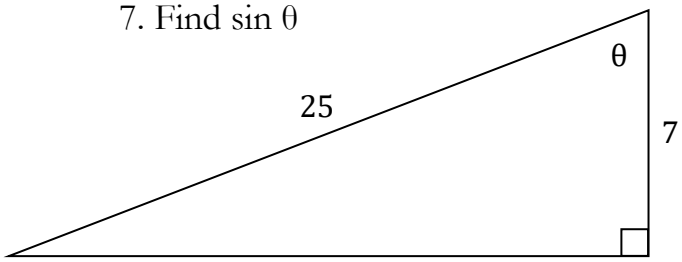
5. Find $\sin\theta$



6. Find $\tan\theta$



7. Find $\sin\theta$



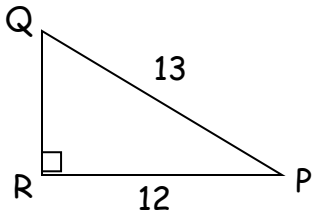
Reciprocal Functions	
$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$
$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

Also Important
$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$$\frac{\sin\theta}{\cos\theta} = \frac{opp/hyp}{adj/hyp} = \frac{opp}{adj} = \tan\theta$$

Find the missing side of the triangle below and give the value of each trigonometric ratio.

1. $\sin(P) =$ _____
2. $\sin(Q) =$ _____
3. $\cos(P) =$ _____
4. $\tan(P) =$ _____



5. $\cot(Q) =$ _____
6. $\sec(P) =$ _____
7. $\csc(P) =$ _____
8. $\tan(Q) =$ _____

9. Given the cos(θ) = 3/5, find the other five trigonometric ratios.

sin(θ) =_____ tan(θ) =_____ csc(θ) =_____

sec(θ) =_____ cot(θ) =_____

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Week 6

Domain: *Number and Quantity; Expressions and Equations*

Standards:
N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.

Simplifying Radicals

When working with the simplification of radicals, you must remember some basic information about ***perfect square*** numbers.

You need to remember:

Perfect Squares
4 = 2 x 2
9 = 3 x 3
16 = 4 x 4
25 = 5 x 5
36 = 6 x 6
49 = 7 x 7
64 = 8 x 8
81 = 9 x 9
100 = 10 x 10

Square roots
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$
$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$

While there are certainly many more perfect squares, the ones appearing in the charts above are the ones most commonly used.

To simplify means to find another expression with the same value. It does not mean to find a decimal approximation.

1. Find the **largest** perfect square which will divide evenly into the number under your radical sign. This means that when you divide, you get no remainders, no decimals, and no fractions.

Reduce: $\sqrt{48}$ to the **largest** perfect square that divides evenly into 48 is **16.**

If the number under your radical cannot be divided evenly by any of the perfect squares, your radical is already in simplest form and cannot be reduced further.

2. Write the number appearing under your radical as the product (multiplication) of the perfect square and your answer from dividing.

$$\sqrt{48} = \sqrt{16 \cdot 3}$$

3. Give each number in the product its own radical sign.

$$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3}$$

4. Reduce the "perfect" radical which you have now created.

$$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$$

5. You now have your answer.

$$\sqrt{48} = 4\sqrt{3}$$

Simplify: $3\sqrt{50}$

Don't let the number in front of the radical distract you.
It is simply "along for the ride" and will be multiplied times our final answer.

The largest perfect square dividing evenly into 50 is 25.

$$3\sqrt{50} = 3\sqrt{25 \cdot 2} = 3\sqrt{25}\sqrt{2}$$

Reduce the "perfect" radical and multiply times the 3 (who is "along for the ride")

$$3\sqrt{25}\sqrt{2} = 3 \cdot 5\sqrt{2} = 15\sqrt{2}$$

Simplify the following.

a) $\sqrt{28} =$

b) $\sqrt{50} =$

c) $\sqrt{45} =$

d) $\sqrt{98} =$

e) $\sqrt{48} =$

f) $\sqrt{300} =$

g) $\sqrt{150} =$

h) $\sqrt{80} =$

Reduce to lowest terms.

a) $\frac{\sqrt{20}}{2} =$

b) $\frac{\sqrt{72}}{3} =$

c) $\frac{\sqrt{22}}{2} =$

Simplify the following expressions.

$$\frac{6 \pm 2\sqrt{45}}{6}$$

$$\frac{10 \pm 3\sqrt{12}}{4}$$

$$\frac{5 \pm \sqrt{75}}{10}$$

$$\frac{3 \pm \sqrt{121}}{4}$$

$$\frac{2 \pm 4\sqrt{7}}{6}$$

$$\frac{2 \pm 6\sqrt{14}}{4}$$

$$\frac{8 \pm 4\sqrt{12}}{16}$$

$$\frac{1 \pm \sqrt{32}}{4}$$

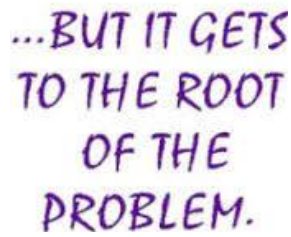
$$\frac{4 \pm \sqrt{28}}{6}$$

$$\frac{7 \pm \sqrt{98}}{7}$$

$$\frac{9 \pm \sqrt{54}}{6}$$

$$\frac{12 \pm \sqrt{200}}{10}$$

Week 7

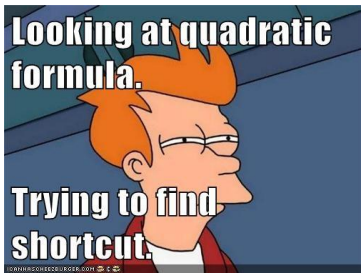


Domain: *Algebra*

Standards:

A.REI.4 Solve quadratic equations in one variable.

A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.



The quadratic formula can be used to solve quadratic equations that cannot be solved by factoring over the integers. The quadratic formula can be very useful in solving problems. Thus, it should be practiced enough to develop accuracy in using it and to allow you to commit the formula to memory.

Quadratic Expression	**Quadratic Equation**	**Quadratic Formula**
$ax^2 + bx + c$	$ax^2 + bx + c = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Using the Quadratic Formula:	Shortcut – To limit mathematical errors and shorten steps when using the quadratic formula:
1) Equation must be in quadratic form. 2) Identify a, b, and c. 3) Input values into the formula and simplify. $2x^2 - 3x - 10 = 0$ a =2, b=-3, c =-10 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-10)}}{2(2)}$ $x = \frac{3 \pm \sqrt{9+80}}{4} = \frac{3 \pm \sqrt{89}}{4}$	1) Identify a, b, and c. 2) Find $b^2 - 4ac$. 3) Input values into the formula and simplify. $2x^2 - 3x - 10 = 0$ a =2, b=-3, c =-10 $b^2 - 4ac = (-3)^2 - 4(2)(-10) = 89$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-3) \pm \sqrt{89}}{2(2)} = \frac{3 \pm \sqrt{89}}{4}$

Solutions: $x = \frac{3 + \sqrt{89}}{4}$ and $x = \frac{3 - \sqrt{89}}{4}$

Approximate solutions for this equation are 3.1085 and -1.6085.

Use the quadratic formula to solve each of the following quadratic equations. Approximate your solutions to two decimal places.

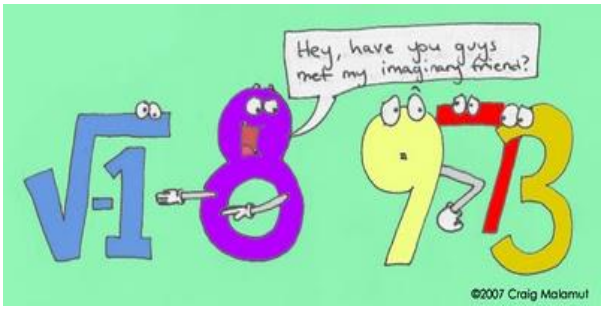
Quadratic Equations	Identify a, b, c	Find $b^2 - 4ac$	Substitute into $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x^2 - 7x + 10 = 0$			
$x^2 - 14x + 45 = 0$			
$0.25x^2 - 0.25x - 10.5 = 0$			
$4x^2 + 8x = 77$			
$x^2 - 11x + 30.25 = 0$			
$3 = 16p - 20p^2$			
$x^2 = 15 + 2x$			
$x^2 - 10x + 35 = 7x - 35$			
$2x^2 + 5 + 20x = 20 - 8x^2 + x$			

Algebra II – Unit 1 Preview

Domain: *Number and Quantity*, Algebra

Standards:

N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.
A.CED.1 Create equations that describe numbers or relationships.
A.SSE.1 Interpret the structure of expressions.



<u>Imaginary Number</u>
An Imaginary Number, when squared, gives a negative result: $i^2 = -1$; The "unit" Imaginary Number (the equivalent of **1** for Real Numbers) is $\sqrt{-1}$ (the square root of minus one).

If ***a*** is a positive real number, then the principal square root of negative ***a*** is the imaginary number $i\sqrt{a}$; that is, $\sqrt{-a} = i\sqrt{a}$.

Example 1: $\sqrt{-7} = \sqrt{7} \cdot \sqrt{-1} = i\sqrt{7}$

Example 2: $\sqrt{-9} = \sqrt{9} \cdot \sqrt{-1} = 3i$

Example 3: $\sqrt{-99} = \sqrt{9} \cdot \sqrt{11} \cdot \sqrt{-1} = 3 \cdot i \cdot \sqrt{11} = 3i\sqrt{11}$

Simplify each expression.

1. $\sqrt{-16}$
2. $\sqrt{-28}$
3. $\sqrt{-5}$
4. $\sqrt{-10} \bullet \sqrt{-2}$
5. $\dfrac{\sqrt{-50}}{\sqrt{-10}}$
6. $\sqrt{64-100}$

<u>Complex Number</u>

A *complex number* is a number in the form $a+bi$, where a and b are real numbers and $i=\sqrt{-1}$. The number a is the real part of $a+bi$ and bi is the imaginary part. Any solution should be written in this form.

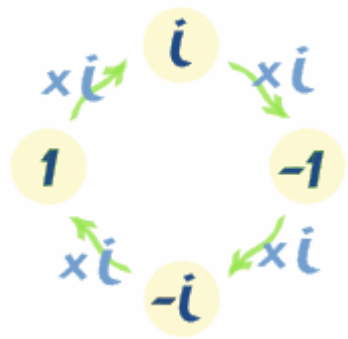
So, a Complex Number has a real part and an imaginary part.

But either part can be **0**, so all Real Numbers and Imaginary Numbers are also Complex Numbers.

Complete the table.

Complex Number	Real Part	Imaginary Part
3 + 2i	3	2
5	5	**0**
-6i	**0**	-6
1 + 7i		
5.2i		
4		
2 - 5i		
2 + πi		
8.2 – 5.5i		
$\sqrt{2}+\frac{i}{2}$		
$\sqrt{-1}$		

Patterns of *i*



The Unit Imaginary Number, ***i***, has an interesting property. It "cycles" through 4 different values each time you multiply. To determine the value of ***i*** to any power, just find the largest multiple of 4 and ***i*** to that multiple of 4 =1, then the remainder is the exponent of ***i*** that is evaluated and multiplied by 1.

Example: $i^{347} = i^{344} i^{3} = (1)(-i) = -i$.

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

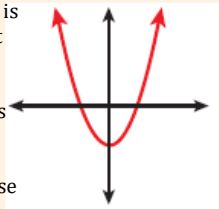
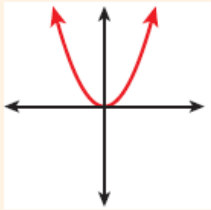
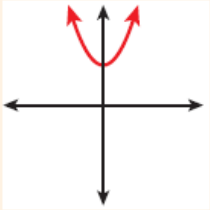
$$i^n = \begin{cases} 1 & \text{if the remainder is 0} \\ i & \text{if the remainder is 1} \\ -1 & \text{if the remainder is 2} \\ -i & \text{if the remainder is 3} \end{cases}$$

Evaluate:

1. i^{30}
2. i^{210}
3. $3i^7$
4. $-25i^{25}$
5. $i + i^3$
6. $i^{19} - i^{16}$
7. $3i^4 + 5i^6$
8. $i^{37} \div i^{35}$
9. $i^{102} \bullet i^{-98}$
10. $\left(i^2\right)^3$

Discriminant

The discriminant of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ is $b^2 - 4ac$.

$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
two distinct real solutions	one distinct real solution	two distinct nonreal complex solutions
If b^2-4ac is a perfect square, then solutions are rational. Otherwise they are irrational. 	 Produces two equal rational roots	

Complete the table by calculating the discriminant to determine the number and nature of the solutions of the following quadratic equations.

Quadratic	Identify a, b, c	Discriminant (b^2-4ac)	Number of Solutions	Nature of Solutions
$y = x^2 + 4$	**1, 0, 4**	**-16**	**2**	**non-real, complex**
$y = x^2$	**1, 0, 0**	**0**	**1**	**real, rational**
$y=-x^2- 5x - 50$				
$-6x^2 - 6 = -7x - 9$				
$4k^2 + 5k + 4 = -3k$				
$-7n^2 + 16n = 8n$				
$2x^2 = 10x + 5$				
$-9b^2 = -8b + 8$				
$5b^2 + b - 2 = 0$				
$-2x^2 - x - 1 = 0$				
$-4m^2 - 4m + 5 = 0$				
$y=-x^2 - 5x + 50$				
$y=-x^2 -4x - 4$				
$8b^2 - 6b + 3 = 5b^2$				
$9n^2 - 3n - 8 = -10$				

Critical Thinking Questions:

a) Write a quadratic equation that has two imaginary solutions.

b) In $y=2x^2-4x+k$, find k that will produce 2 equal real roots. Explain how you determined your answer.

c) In your own words, explain why it is impossible for a quadratic equation to have one imaginary solution.